

# Dimensioning of Communication Links for Base-station and MSC Interconnection in CDMA Mobile Communication Systems

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## Abstract

*In CDMA mobile communication systems, voice is encoded and packaged in variable length packets that are transported between the mobile station and the switching center. While the packetization provides a great flexibility in resource allocation, it poses a QOS challenge on voice. In this paper, we discuss link dimensioning for a typical CDMA system. We consider a communication link connecting a CDMA base station and Mobile Switching Center. We analyze the resulting queueing system using large deviations theory and provide results for the minimum link capacity needed to support a given number of CDMA voice sources. Our results show the potential gain that can be achieved by statistical multiplexing of voice sources and discuss the effects of the statistics of the voice process on the multiplexer performance.*

## 1. Introduction

Code Division Multiple Access (CDMA) cellular system promises many advantages over the AMPS and TDMA systems. These advantages include enhanced privacy, resistance to jamming, improved voice quality, improved handoff performance, and soft (and increased) capacity [1]. One important aspect of CDMA is the usage of a variable bit rate (VBR) voice encoder which reduces the required bandwidth and interference. The vocoder detects speech and silence in the voice process and adjusts its rate accordingly. It also tunes out background noise and dynamically varies its data transmission rate to operate at one of four different levels.

This VBR vocoder impacts the airlink interface capacity as shown in [2]. Though the air link capacity is the scarce resource in a cellular system, it is nonetheless important to optimize the usage and design the land

interconnection network efficiently. A base station (BS) is usually connected to a central switching office (MSC) via leased lines. These leased lines are usually quite expensive and add to the cost of operating the cellular system.

In this paper we study the issue of BS-MSC interconnection. We provide a methodology for performance analysis and dimensioning of the involved communication links to satisfy the required quality of service. We use large deviations theory techniques as developed in [3,4] to analyze the problem.

The rest of this paper is organized as follows. In section 2, we describe the problem under consideration and state the objectives of studying it. In section 3, we present the analysis of the model. In section 4, we provide a numerical study and validation of the models introduced. Section 5 concludes the paper.

## 2. Problem Description

Consider a base station in a CDMA system. The major functionality of the BS is to perform the IS-95 air interface specifics and provide connection between the mobile users and the central office switch. Let us focus our attention on the BS--MSC link in the reverse direction (i.e. from BS to MSC). Let the link capacity be  $C$  bits/sec. The link is statistically shared among both the packetized voice traffic of the connected sources and the signaling packets. A limited buffer of size  $B$  bits is provided to store the incoming packets until the link becomes available.

Let us now describe the sequence of operations performed on voice packet until they reach the link buffer. At the mobile, the vocoder adapts its rate according to speech activity, noise and threshold. In steady state, an 8K vocoder is in one of four rates with a probability as shown in Table 1. Another version of the vocoder that provides better voice quality operates at a maximum rate of 14.4 kbps (13K vocoder). In this work, we focus on the 8K vocoder without loss of generality. At the base station, each

mobile is allocated a digital signal processor unit that performs IS-95 processing and retrieves the packetized voice data from the raw IS-95 stream. . A system wide frame of  $T=20$  msec is used to multiplex the packets. The frame is divided into  $M$  slots each with length  $T/M$  msec. Each voice source is assigned one slot during call setup and is only allowed to transmit packets at the assigned slot. This slot may change only when a call goes through a hard handoff. The slot assignment is done such that the load is distributed evenly among the slots. The link buffer receives the packetized voice and signaling packets in its common buffer and transmits the packets in a FIFO manner.

Table 1: Rate distribution for a 8K vocoder

Rate in bps	Packet length	Probability
9600	256	0.291
4800	160	0.039
2400	120	0.072
1200	96	0.598

The quality of service for the BS--MSC link is defined as follows:

1. Maximum delay for an arbitrary packet should be less than  $d$  msec. Typically,  $d=4$  msec. Since the delays involved are random, we express the maximum delay as the 99.99% of delays of the delay distribution of an arbitrary packet
2. The packet loss probability due to the finite buffer should be kept below  $\epsilon$ , where  $\epsilon$  is typically  $10^{-6}$  to  $10^{-3}$

The questions we need to address are: 1) for a given number of voice channels,  $N$ , what is the minimum link speed  $C$  required? 2) for a given link capacity  $C$ , what is the number of voice channels  $N$  that can be supported? 3) What is a good size for the buffer  $B$  to sustain the required quality of service. Large buffers would increase maximum possible delay while enhancing the loss performance. Signaling traffic is assumed to generate  $\alpha\%$  of the voice traffic. Typically  $\alpha$  is about 1% to 10%. We concentrate on the voice traffic only and we can scale the obtained results to reflect the effect of signaling .

### 3. Analysis of the Queuing Problem

Large deviations theory provides techniques for estimating properties of rare events such as their frequency and the manner in which they occur. Recently, large deviations theory gained popularity as a valid methodology for analysis of emerging ATM networks [3,5,6]. Elwalid et al. [7] proposed the Chernoff-Dominant Eigenvalue approximation to model the buffering behavior of some queuing systems in the asymptotic case as follows. The queue length distribution of a queuing system fed by a large number of Markov-modulated sources is approximated by:

$$G(b) \approx A e^{zb}$$

where  $G(b)$  is the cumulative distribution function that the queue length is larger than  $b$ ,  $A$  is the loss in a bufferless multiplexing system as estimated from Chernoff theorem, and  $z$  is the dominant eigenvalue in buffered multiplexers that determines the large buffer behavior. In the following we summarize these results: Consider  $K$  classes of sources where each class  $k$  is comprised of  $N_k$  sources characterized by  $(Q^{(k)}, R^{(k)})$  where  $Q^{(k)}(i, j), i \neq j$  is the rate at which a source in state  $i$  jumps to state  $j$  and  $Q^{(k)}(i, i) = -\sum_{j \neq i} Q^{(k)}(i, j)$ . The vector

$R^{(k)} = (R_1^{(k)}, R_2^{(k)}, \dots, R_d^{(k)})$ , where  $R_i$  is the rate at which a source in state  $i$  generates traffic. Let  $V_{k,i}(t)$  be the rate of traffic generated by source  $i$  of class  $k$  at time  $t$ . Let  $\{V_{k,i}\}$  be the stationary distribution of  $V_{k,i}(t)$ . Let  $V = \sum_k \sum_i V_{k,i}(t)$ , then in a buffer-less system, loss occurs when  $V > C$ . Therefore, we estimate  $P(V > C)$ . Let  $\pi^{(k)}$  be the stationary probability vector of class  $k$  sources then  $V_{k,i}$  has the moment generating function

$$M_k(s) = E(e^{sV_{k,i}}) = \sum_i \pi_i^{(k)} e^{sR_i^{(k)}}$$

Chernoff theorem states that

$$\log P(V > C) \leq -F(s^*)$$

where  $F(s) = sC - \sum_k N_k \log M_k(s)$  and  $F(s^*) = \sup_{s \geq 0} F(s)$ .

Hence,  $A = \exp(-F(s^*))$ . The dominant eigenvalue of the system is obtained as follows. Consider a source  $(Q, R)$  and let  $R_d = \text{diag}(R_1, R_2, \dots, R_n)$ , for  $z$  real and negative, the matrix  $(R_d - \frac{1}{z}Q)$  is an irreducible matrix with non-negative off-diagonal elements. This matrix has a real eigenvalue, called the maximum real eigenvalue that is greater than the real part of all the other eigenvalues. Let  $g(z) = \text{MRE}(R_d - \frac{1}{z}Q)$ . For  $K$  classes of heterogeneous sources, the dominant eigenvalue  $z$  is obtained by solving the equation:

$$\sum_{k=1}^K N_k g^{(k)}(z) = C$$

where  $g^{(k)}(z) = \text{MRE}(R_d^{(k)} - \frac{1}{z}Q^{(k)})$ , see [5] for details.

### Discrete-Time Systems

For discrete-time Markov sources, the calculation for  $A$  does not change, however  $z$  is calculated differently. Let a source be characterized by  $(P, R)$  where  $P$  is the probability transition matrix governing the underlying Markov chain of

the source. Let  $\mu(z)$  be the Perron-Frobenius eigenvalue of  $e^{-zR_d} P$ . For real  $z$ , the matrix  $e^{-zR_d}$  is non-negative and irreducible, hence its PF-eigenvalue is real, positive, and simple. We thus have  $z$  as the unique solution of

$$\sum_{k=1}^K N_k \left\{ -\frac{\log \mu^{(k)}(z)}{z} \right\} = C$$

### Approximation of the CDMA Link Multiplexing using Chernoff-Dominant Eigenvalue Approach

First we need justify the use of large deviations theory in this context. The Erlang capacity of a 3-sector base station will be in the range of 36-72 [2]. Thus on the average the system sees a large number of connected voice users at any time (including those in handoff). The first approximation we take is that we assume that the framing structure imposed on the voice sources (see section 2) can be ignored in the long term. The second approximation performed is that we observe from Table 1, that the source is mainly in full rate (9.6 kbps) or one eighth rate (1.2 kbps). We assume that the half and one fourth rate are aggregated into the full rate state and their probabilities are added to the probability of being in full rate. The sources we get are therefore a worst case source w.r.t the actual source. This would allow simplification and reduction of the calculations needed to evaluate  $A$  and  $z$ . The resulting source is a two-state source with state 1 having a steady state probability of  $\pi_1=0.598$  and rate  $R_1=4800$  bits/sec and state 2 with a steady state probability of  $\pi_2=0.402$  and rate  $R_2=12800$  bits/sec.

### Discretization of the Problem

At the full rate, let the maximum packet size in bits generated over a 20-msec period be  $P_{\max}$  and let  $P_{\min}$  be the minimum packet size in bits generated over the same period in the one-eighth rate. Define  $P_{\text{cell}} = \text{gcd}(P_{\min}, P_{\max})$ . We artificially introduce small packets of size  $P_{\text{cell}}$  which we call *cells*. An incoming packet is (artificially) divided into a number of cells. In the worst case,  $P_{\text{cell}}$  is equal to one bit. Let  $\tau$  be the transmission time of a cell on the link speed with rate  $C$ . The buffer size is expressed in terms of the cells as follows,  $\hat{B} = B / P_{\text{cell}}$ . The link speed  $C$  is normalized such that the multiplexer is capable of transferring one cell each  $\tau$  time units. The rates at full and one eighth states are normalized with respect to the new link speed. Let us denote the one-eighth and full rate state as states 1 and 2. Let the normalized rates at these states be denoted by  $\hat{r}_1$  and  $\hat{r}_2$  respectively, then  $\hat{r}_i = r_i / C$ ,  $i=1,2$ . Furthermore, let the mean length of the sojourn time of the

source in state  $i$  be  $\bar{t}_i$  cells. We are now in a position to show the results for our system.

Using the above results we have a homogenous set of  $N$  sources each characterized by a 2-state Markov chain. The probability transition matrix of the source is given by  $P = \begin{bmatrix} \alpha_1 & 1-\alpha_1 \\ 1-\alpha_2 & \alpha_2 \end{bmatrix}$ , where  $\alpha_i = 1 - 1/\bar{t}_i$ ,  $i=1,2$  and its rate vector  $R$  is given by  $R = (\hat{r}_1, \hat{r}_2)$ . We apply the Chernoff-dominant eigenvalue approach as follows.

### Finding the value of A

For the purpose of evaluating the constant  $A$ , the 2-state CDMA voice source can be mapped to an on-off source as follows. Since  $\hat{r}_1 < \hat{r}_2$ , for a given number of sources  $N$ , there is always at least  $N \times \hat{r}_1$  data units out of the link speed being utilized by the sources. We can use the following equivalent system for the purpose of calculating the value of  $A$ . If we use a link with capacity  $\tilde{C} = 1 - N \times \hat{r}_1$  with  $N$  on-off sources where the rate of the on state is equal to  $\tilde{r}_2 = \hat{r}_2 - \hat{r}_1$  and the rate of the off state is of course  $\tilde{r}_1 = 0$ .

Weiss [4] reports that for such a system of  $N$  on-off sources with link speed  $\tilde{C}$  and  $N$  sources, the value of  $A$  is given by:

$$A = \exp \left( -N \left[ \tilde{c} \ln \left( \frac{\tilde{c}}{\pi_1} \right) + (1-\tilde{c}) \ln \left( \frac{1-\tilde{c}}{1-\pi_1} \right) \right] \right)$$

$$\text{where } \tilde{c} = \frac{\tilde{C}}{N\tilde{r}_2}$$

### Finding the value of z

To find the value of the dominant eigenvalue of the system we solve for  $z$  which is a solution of  $N \left( -\frac{\log \mu(z)}{z} \right) = C$

where  $\mu(z)$  is the PF-eigenvalue of the matrix  $X(z) = e^{-zR_d} P$ . We have

$$X(z) = \begin{bmatrix} e^{-r_1 z} \alpha_1 & e^{-r_1 z} (1-\alpha_1) \\ e^{-r_2 z} (1-\alpha_2) & e^{-r_2 z} \alpha_2 \end{bmatrix}. \quad \text{To find the}$$

eigenvalues of  $X(z)$ , we solve the equation  $|\mu I - X(z)| = 0$ . This gives the PF-eigenvalue equal to

$$\mu(z) = \frac{1}{2} \left\{ \alpha_1 e^{-r_1 z} + \alpha_2 e^{-r_2 z} + \sqrt{(\alpha_1 e^{-r_1 z} + \alpha_2 e^{-r_2 z})^2 - 4(1-\alpha_1-\alpha_2)e^{-(r_1+r_2)z}} \right\}$$

from which we obtain the value of  $z$  that verifies the equation  $N(-\frac{\log \mu(z)}{z}) = C$  via a numerical iteration. As a first approximation to  $z$ , we can use the value of  $z$  for the continuous time fluid flow model which can be expressed in a closed form (see Appendix A).

Once the value of  $A$  and  $z$  is found, then the packet loss probability  $PLP$  is upper-bounded by the cell overflow probability which is approximately equal to  $G(\hat{B}) \approx Ae^{z\hat{B}}$ , i.e.  $PLP \leq Ae^{z\hat{B}}$ . To find the delay bound we note that the probability that the delay can be expressed in terms of the cell transmission time  $\tau$ , so the probability that the delay is larger than  $x\tau$  is given by  $P(\text{delay} \geq x\tau) \approx Ae^{zx}$ . So if we would like to find the value of the delay  $D$  such that  $u\%$  of delays is less than  $D$ . We solve for  $x\tau \approx \ln(1-u)/A$  and we set  $D = \text{maximum}(0, x\tau)$ .

#### 4. Results

In this section we provide some numerical results for the system under study. We set the maximum allowable delay over the link to be 4 msec and the maximum packet loss to be  $10^{-6}$ . We find the buffer overflow in terms of the *cells*. Since a voice packet consists of one or more cells, then the cell loss probability would provide an upper bound on the packet loss since one or more cells that are lost *may* contribute to the loss of the *same* packet. The buffer size has a maximum capacity for a total of 16000 bits of data. Let us consider the following case. The mean time spent in state 1,  $t_1$ , is equal to 600 msec. The mean time in state 2,  $t_2$ , is found using the relation  $t_2/(t_2 + t_1) = \pi_2$ . We vary the link capacity,  $C$ , from 4 DS0 to 32 DS0 channels and for each link capacity we get the maximum number of sources that can be supported without violating the requested QOS parameters. We define the statistical multiplexing gain,  $G$ , as follows.  $G = \frac{\text{sources obtained by stat muxing}}{\lfloor C / \text{peak rate} \rfloor}$  where

$$1 \leq G \leq \frac{\text{Peak Rate}}{\text{Average Rate}} = 2.6667. \text{ In figure 1, we plot the}$$

number of statistically multiplexed sources as a function of the link speed and the corresponding statistical multiplexing gain. We observe the uniform increase in the achieved gain as the link speed increases as is always the case with packet multiplexers. In the range of link speeds studied here, the statistical multiplexing gain does not reach its maximum achievable rate of 2.6667 due to the violation of the packet loss probability. An increase in the buffer size would allow more statistical gain to be achieved. In a related work,

Gerlich et al. [8] studied the same problem using a different approach without ignoring the framing structure imposed on the voice packets. Their model however does not capture the long term correlation in the superposition of multiple voice sources. Compared with [8], we note that for link speeds greater than 16 DS0 channels, the large deviations model presented in this paper allows less sources to be multiplexed while for link speeds less than 8 DS0 channels, it allows more sources and in the middles the two models are very close. We believe that the best approach would be to combine the two effects: the correlation effect and the framing effect.

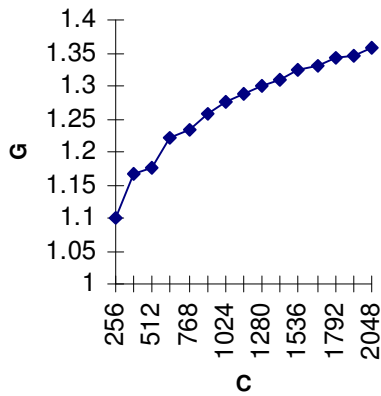
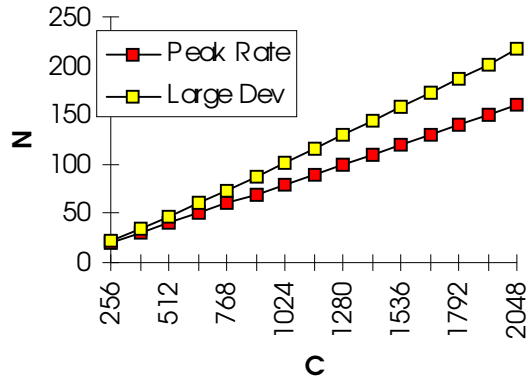
In figure 2, we study the effect of the statistics of the voice process on the multiplexer performance. We consider two cases, one where the link speed is 24 DS0 channels (1544 kbps) and the other for link speed equal to 12 DS0 channels (768 kbps). The mean time in state 2 (peak rate state),  $t_2$ , is varied from 100 to 2000 msec. We observe that the larger the value of  $t_2$ , the smaller the number of sources allocated due to the effects of increased burstiness and correlation in the input process to the multiplexer. We note, however, that the dependence on  $t_2$  is not very high. For example, for  $C = 1544$  kbps, as  $t_2$  increases from 100 msec to 2000 msec, the number of sources decreases from 173 to 160 or 7.51%. For  $C = 768$  kbps, as  $t_2$  increases from 100 msec to 2000 msec, the number of sources decreases from 81 to 75 or 7.41%. The important conclusion of this is that the results reported here are not very dependent on the time spent in the states and its statistics which we have assumed to be  $t_2=600$  msec and which is known to be valid for other voice encoding schemes other than the CDMA particular encoding scheme.

#### 5. Conclusions

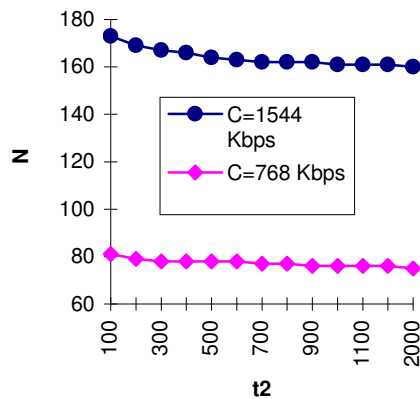
CDMA voice encoding and statistical multiplexing provide a means for making the most out of the communication links interconnecting base stations and MSC. We introduced a methodology based on large deviations theory that can be used to study the performance of these links. We have introduced some approximations of the original problem and ignored the framing imposed on the voice packets in the base station. The resulting system is easy to analyze and provides insight on the system behavior due to the correlation in the voice process.

The work presented here should be combined with the work reported by Gerlich et al. [8] in order to provide a safer scheme for statistical multiplexing of CDMA encoded voice. The approach should decompose the problem into two time scales: one based on the framing of voice packets and the second based on the voice encoder rate process and

its statistics in a manner similar to the method developed in this paper.



**Figure 1:** The number of sources versus link capacity and the corresponding statistical gain



**Figure 2:** Effect of the mean time in state 2 (peak rate) on the number of sources that can be statistically multiplexed without violating the QoS for C=1544 Kbps and 768 Kbps

## Appendix

For a 2-state Markov fluid source (continuous-time case)

described by  $(Q, R)$ , where  $Q = \begin{bmatrix} -\alpha_1 & \alpha_1 \\ \alpha_2 & -\alpha_2 \end{bmatrix}$  and

$R = (\hat{r}_1, \hat{r}_2)$ , the value of the dominant eigenvalue of the system is given by  $z = \frac{\hat{c}(\alpha_1 + \alpha_2) - (\alpha_2 \hat{r}_1 + \alpha_1 \hat{r}_2)}{\hat{c}^2 - \hat{c}(\hat{r}_1 + \hat{r}_2) + \hat{r}_1 \hat{r}_2}$  where

$\alpha_i = 1/t_i$ ,  $\hat{c} = C/N$ , and  $\hat{r}_i = r_i/C$ . This can be easily verified by finding  $g(z) = \text{MRE}(R_d - \frac{1}{z}Q)$  and solving for  $g(z) = C/N$ .

## References

- [1] A. M. Viterbi. *CDMA Principles of Spread Spectrum Communication*. Addison--Wesley, Reading, MA, 1995.
- [2] A. M. Viterbi and A. J. Viterbi. Erlang Capacity of a Power Controlled CDMA System. *IEEE Journal on Selected Areas in Communications*, 11(6):892--899, August 1993.
- [3] A. I. Elwalid and D. Mitra. Effective Bandwidth of General Markovian Traffic Sources and Admission Control of High Speed Networks. *IEEE Transactions on Networking*, 1:329--343, 1993.
- [4] A. Weiss, An Introduction to Large Deviations Theory for Communications Networks, *IEEE Journal on Selected Areas in Communications*, 13:938--952, 1995.
- [5] G. Kesidis, J. Walrand, and C.-S. Chang. Effective Bandwidths for Multiclass Markov Fluids and Other ATM Sources. *IEEE Transactions on Networking*, 1(4):424--428, August 1993.
- [6] W. Whitt. Tail Probabilities with Statistical Multiplexing and Effective Bandwidth for Multi-Class Queues. *Telecommunications Systems*, 2:71--107, 1993.
- [7] A. Elwalid, D. Heyman, T. V. Lakshman, D. Mitra, and A. Weiss, Fundamental Bounds and Approximations for ATM Multiplexers with Applications to Video Teleconferencing. *IEEE Journal on Selected Areas in Communications*, 13:1004--1016, 1995.
- [8] N. Gerlich, K. Elsayed, P. Tran-Gia, and N. Jain, Performance Analysis of Link Carrying Capacity in CDMA Systems, Accepted for publication in *ITC'14*.