Statistical Multiplexing with Arbitrary On/Off Sources

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Abstract

We develop a methodology for characterizing the superposition process of $N \geq 2$ discrete-time arbitrary on/off sources. The superposition is a discrete-time semi-Markov process with $O(2^{2N})$ states. We use the superposition model to analyze a finite-buffer statistical multiplexer with multiple arbitrary on/off input sources. We study the effect of various traffic parameters on the queueing performance.

Keywords
statistical multiplexing - ATM - on/off sources - superposition processes - semi-Markov processes

1 Introduction

In an ATM environment many types of traffic, such as voice, data, and video, are to be efficiently transported by the same network. An ATM multiplexer receives cells (fixed size packets of 53 octets length) from a number of different incoming links and then transmits them out onto a single outgoing link. A finite buffer is provided in the multiplexer to accommodate the multiple arrivals of cells. Each arrival stream is modeled by a bursty and possibly a correlated process. The service time is deterministic and is equal to one slot of the outgoing link which is assumed to be long enough to transmit one cell.

The analysis of such a queueing system is quite complex due to the large number of arrival processes. A possible method for approximately analyzing the queue is to first characterize the superposition process of all arrival processes, and then analyze the queue with a single arrival process. The problem of characterizing the superposition process of a set of arrival processes has been addressed extensively in the literature. One approach for obtaining the superposition process is to approximate it by a renewal process, see Albin [1], Whitt [21], Sriram and Whitt [17], and also Perros and Onvural [15]. Heffes and Lucantoni [10] considered the superposition process of packetized voice sources. They approximated the superposition by a Markov Modulated Poisson Process (MMPP). The accuracy of the model is reasonable when the average delay in the multiplexer is the amount of paramount importance. However, it did not provide a good estimate for the packet loss probability. Several other authors (see [3, 13, 20]) suggested alternative methods for characterizing the superposition process of on/off sources as an MMPP in order to improve the accuracy with regards to calculation of the cell loss probability. Also, Heffes [9] obtained an MMPP approximation to the superposition of different MMPP arrival processes using a set of simple expressions.

An alternative method to analyze statistical multiplexers is the Uniform Arrival and Service (UAS) model (also known as fluid flow). In this case an on/off process produces a uniform flow of bits when in the on state. Cell departures are modeled as a uniform flow out of the queue. Anick, Mitra, and Sondhi [2] evaluated the system performance using simple expressions for a multiplexer with infinite buffer space and homogeneous arrival processes. Tucker [19] considered the finite buffer case. The methodology was generalized to the case of heterogeneous Markov Modulated Rate Processes in a series of papers [18, 7, 6].

In this paper we develop a methodology for characterizing the superposition process of $N \geq 2$ discrete-time arbitrary on/off sources. The superposition is a discrete-time semi-Markov process with $O(2^{2N})$ states. The major advantage of this methodology is that it provides a uniform framework in which a variety of traffic types can be handled. We use the superposition model to analyze a finite-buffer statistical multiplexer with multiple input arbitrary on/off sources. We study the effect of various traffic parameters on the queueing performance. The results obtained show the need for capturing the effect of the distribution of the on and off periods in call admission control and bandwidth allocation.

2 The Arbitrary On/Off Source

A popular model for traffic processes in an ATM environment is the on/off source. It is used extensively for modeling voice as well as other types of traffic. A common practice is to assume that the on and off periods have a geometric or exponential distribution to simplify the analysis. This assumption may not be adequate for many practical cases as shown by traffic measurements [14]. Moreover, earlier results by Kosten [12] indicate that even in the asymptotic case when the number of sources approach infinity, the effect of the periods distribution does not vanish.

We consider the discrete-time arbitrary on/off source as a versatile traffic model in which the on and off periods distribution is allowed to have an arbitrary discrete probability density function (PDF). Our goal is to study the effects of the distribution of the periods on the performance of a statistical multiplexer in an ATM network. In our model, successive on and off periods are independent and are mutually independent. For source i, the distributions of the off and on periods are specified by the arbitrary probability density functions $f_i^{Off}(k)$ and $f_i^{On}(k)$ respectively. In this paper we consider the special case when all sources emit one ATM cell per time slot during the on period and assume that all sources have the same slot size (speed).

2.1 The Superposition of Multiple Arbitrary On/Off Sources

In [5], we introduced a methodology for approximately characterizing the superposition process of multiple discrete-time semi-Markov processes in terms of yet another semi-Markov process. We provided an algorithmic procedure for computing the semi-Markov kernel of the superposition process. It is clear that an arbitrary on/off source is in essence an alternating renewal process which can be characterized as a semi-Markov process. It is therefore possible to model an arbitrary on/off source, say source i, as a special semi-Markov process with state space $\{0, 1\}$, where 0 and 1 denotes the off and on states respectively, and with a semi-Markov kernel

$$\boldsymbol{G}_i(k) = \begin{bmatrix} 0 & f_i^{off}(k) \\ f_i^{on}(k) & 0 \end{bmatrix}.$$

We are now in a position to specialize the results in [5] to the case of arbitrary on/off sources. Consider $N \geq 2$ possibly heterogeneous arbitrary on/off sources. The superposition state is observed at those instants in time when one or more of the component processes change state from on to off or vice versa. The superposition state is described by the tuple $[(x_1, t_1), (x_2, t_2), \cdots, (x_N, t_N)]$ where $x_i \in \{0, 1\}$ is the state of source i and $t_i \in \{0, 1\}$ is such that $t_i = 1$ iff process i changes state. The state space Ξ of the superposition process is given by

$$\Xi = \left\{ \begin{array}{l} [(x_1, t_1), (x_2, t_2), \cdots, (x_N, t_N)] : x_i \in \{0, 1\}, \\ t_i \in \{0, 1\}, \ \sum_{i=1}^{N} t_i \neq 0 \end{array} \right\}$$

The number of states in the superposition is obviously equal to $2^N(2^N - 1)$.

In order to fully characterize the superposition process we must find the the kernel $Q = [q(u, v, k)]$, where $q(u, v, k)$ is the probability that given the superposition is currently at state u and that the next state would be v, the sojourn time would be equal to k, where u and $v \in \Xi$ and $k \geq 1$. In [5], this is given by:

$$q(u, v, k) = \prod_{i=1}^{N} \phi_i(u, v, k)$$

where $\phi_i(u, v, k)$ is calculated as follows. Let $(x_i(s), t_i(s))$ be the state of source i when superposition is at state i. Let $F_i^{off}(k) = \sum_{j=1}^{k} f_i^{off}(j)$ and $F_i^{on}(k) = \sum_{j=1}^{k} f_i^{on}(j)$ be the cumulative probability distribution functions of the off and on periods of sources i respectively. Also, let $\hat{f}_i^{off}(k)$ and $\hat{f}_i^{on}(k)$ be the residual life-time distribution of the off and on periods of source i respectively. Define the associated cumulative probability distribution functions $\hat{F}_i^{off}(k)$ and $\hat{F}_i^{on}(k)$ accordingly. Given values of $t_i(u)$ at the origin state u and $t_i(v)$ at the destination state v, the following four cases are possible:

I- $t_i(u) = t_i(v) = 0$. In this case we have $x_i(u) = x_i(v)$, i.e. source i does not change state in either state u or v. The probability of such an event to occur for source i is approximately given by

$$\phi_i(u, v, k) = \begin{cases} 1 - \hat{F}_i^{off}(k) & \text{if } x_i(u) = 0 \\ 1 - \hat{F}_i^{on}(k) & \text{if } x_i(u) = 1 \end{cases}$$

II- $t_i(u) = 0$ and $t_i(v) = 1$. In this case source i changes state at v but not at u. Since source i has not changed state at u, the probability that this event occurs is

$$\phi_i(u, v, k) = \begin{cases} \hat{f}_i^{off}(k) & \text{if } x_i(u) = 0 \\ \hat{f}_i^{on}(k) & \text{if } x_i(u) = 1 \end{cases}$$

III- $t_i(u) = 1$ and $t_i(v) = 0$. In this case source i changes state at u but not at v. The probability that this event occurs is

$$\phi_i(u, v, k) = \begin{cases} 1 - F_i^{off}(k) & \text{if } x_i(u) = 0 \\ 1 - F_i^{on}(k) & \text{if } x_i(u) = 1 \end{cases}$$

IV- $t_i(u) = t_i(v) = 1$. In this case source i changes state at both u and v. The probability that this event occurs is given by

$$\phi_i(u, v, k) = \begin{cases} f_i^{off}(k) & \text{if } x_i(u) = 0 \\ f_i^{on}(k) & \text{if } x_i(u) = 1 \end{cases}$$

Note that in a state $u \in \Xi$ of the superposition source, we have $a(u)$ arrivals per slot where $a(u) = \sum_{i=1}^{N} x_i$. In [5], we have shown that this model provides an accurate approximation of the actual superposition. Also in [5], a computationally efficient algorithm based on the notion of state aggregation has been developed, but it is not accurate for some types of traffic sources.

3 Analysis of a Statistical Multiplexer with Multiple Arbitrary on/off Input Sources

We consider a FIFO finite buffer multiplexer serving $N \geq 2$ arbitrary on/off sources. The multiplexer has $S \geq 1$ servers and can accommodate a total of $B \geq S$ cells at any time

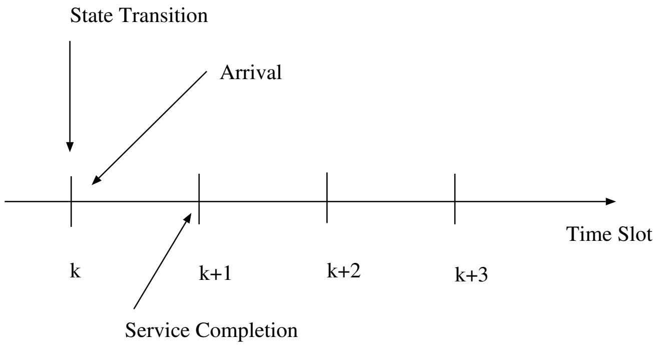


Figure 1: Timing of events in the early arrival model.

instant including those in service. The service time for all cells is constant and is equal to one time slot. The multiplexer can serve S cells every time slot. We assume that $N > S$, otherwise no queue will ever form in the multiplexer and the problem will be trivial to handle. Also, we assume that all sources and the multiplexer output link(s) have the same speed.

We seek the steady state probabilities, $\pi(n)$, $0 \le n \le B - S$, that there are n cells in the multiplexer's queue. From this we can obtain other measures of interest such as the mean queue length, the probability of full buffer and the cell loss probability.

Let us first discuss the timing of events in our system. We follow an *early arrival* timing model as defined by Hunter [11]. That is, during an arbitrary time slot, the sequence of events is as follows: a state transition in the superposition may occur, followed immediately by cell arrivals (if any), which is followed by service of waiting cells if there are any, and finally departure of cells that received service.

The superposition process of the N sources is first characterized as a SMP as described in section 2. Let $M = 2^N(2^N - 1)$ be the number of states of the SMP, with states numbered $0, 1, \cdots, M-1$ and $\boldsymbol{Q} = [q(x, y, k)]$ be its semi-Markov kernel. Let $a(h)$ be the number of active sources at state $h \in \{0, 1, \cdots, M-1\}$ of the superposition process.

<u>System State:</u> The system state at any particular slot is described by the pair (n, h) where $0 \le n \le B - S$ is the number of cells in the multiplexer buffer at the beginning of the slot (not counting the cells that are to arrive at this slot) and $0 \le h \le M - 1$ is the current state of the superposition process. We note here that the process (n, h) at successive time slots does not form a Markov chain. This is due to the non-Markovian nature of the superposition process. However, by observing the system at instants immediately after the superposition process experiences a state transition, the states (n, h) at these instants form an embedded Markov chain since successive states visited by a SMP form a Markov chain.

<u>Solution Method:</u>

- The embedded probability transition matrix $\boldsymbol{P}$ governing transitions between all possible states (n, h) is generated.
- The embedded steady state probabilities $\hat{\pi}(n, h)$ are calculated.
- Finally, the arbitrary point probabilities of observing state (n, h), $\pi(n, h)$, are obtained from $\hat{\pi}(n, h)$.

The two fundamental technical difficulties that arise here are generating the matrix P and the calculation of $\pi(n, h)$ from $\hat{\pi}(n, h)$.

<u>Generation of the Probability Transition Matrix:</u> Consider the queue occupancy evolution process at the multiplexer. Assume that the superposition has just made a transition to state h and that the number of cells in the multiplexer immediately before that transition occurs was n_0. During the time interval at which the superposition process is in state h, $a(h)$ cells arrive at the beginning of each slot. At each time slot, if the number of newly arrived cells plus the number already in the system is greater than B, then the excess cells are dropped randomly. By the end of a time slot a maximum of S cells in the multiplexer (possibly including those who have just arrived) are served. Assume that the superposition is in state h and that it makes a transition to state h' in k slots. Also, assume that the number of cells in the multiplexer when the superposition process made the transition to state h was n_0. Then, the number of cells in the multiplexer after r slots can be calculated using the following recursive equation:

$$n_r = max\left(0, min(n_{r-1} + a(h), B) - S\right), \; r = 1, 2, \cdots \quad (1)$$

By applying the above equation k times we can find the number of cells in the multiplexer k slots after a transition to state h occurs. By conditioning on the probability that superposition makes a transition from state h to state h' in k steps, we *increment* the probability of going from state (n_0, h) to state (n_k, h') by $q(h, h', k)$. The algorithm in figure 2 is used for generating the probability transition matrix $\boldsymbol{P} = \left[p[(n, h), (n', h')]\right]$.

Once the probability transition matrix $\boldsymbol{P}$ is generated, we solve for the invariant probability vector $\hat{\pi}(n, h)$ which is the probability of observing the queueing system in state (n, h) given that the superposition process has just undergone a state transition.

<u>Arbitrary-time Probability Calculation:</u> The key to the calculation of the arbitrary-time probability distribution of the queue occupancy is that the system evolution is deterministic given a specific state h of the superposition process, an initial queue occupancy level n_0, and the number of slots k measured from the instant when the superposition process moved to state h.

Let the state of the system at an instant where a transition occurs be (n_0, h). Let us assume that the superposition process makes a transition to state h' after $k \ge 0$ slots with probability $q(h, h', k)$. Then, all states (n_r, h), $1 \le r \le k-1$, where n_r is calculated using equation 1, will be observed with probability one, conditioned on the initial state (n_0, h) and that a transition from state h to state h' occurs in $l > k$ slots. Probabilities $\pi(n, h)$ can then be calculated using the algorithm shown in figure 3. Note the essential normalization step.

- $\diamond$ Let $p[(n,h),(n^{'},h^{'})] = 0$ for all states.
- $\diamond$ For all states (n_0, h) do
  - $\star$ For all values of k and $h^{'}$
  - $\star$ Find n_k using equation 1
  - $\star$ If $q(h,h^{'},k) \neq 0$ then let $p[(n_0,h),(n_k,h^{'})] = p[(n_0,h),(n_k,h^{'})] + q(h,h^{'},k)$

Figure 2: The algorithm for generating the probability transition matrix.

- $\diamond$ For all states (n,h), let $\pi(n,h) = 0$
- $\diamond$ For all states (n_0,h) do
- $\diamond$ For all possible states h'
  - For all possible values of l
    - If $q(h,h^{'},l) \neq 0$ then
      - for all values of k, $0 < k < l$
        - Find n_k from equation 1
        - Let $\pi(n_k,h) = \pi(n_k,h) + \hat{\pi}(n_0,h)\, q(h,h^{'},k)$
- $\diamond$ Let $\kappa = \sum_{(n,h)} \pi(n,h)$
- $\diamond$ For all states (n,h), let $\pi(n,h) = \frac{\pi(n,h)}{\kappa}$ **(Normalization)**

Figure 3: The algorithm for calculating the arbitrary-time probability

Once the arbitrary point probabilities $\pi(n,h)$ have been found, performance measures of the multiplexer like the mean queue length and the cell loss probability can be obtained. The mean queue length can be easily obtained from the probabilities $\pi(n,h)$. The probability of loss $P(Loss)$ is calculated as follows:

$$P(Loss) = \frac{\sum_n \sum_h min(n + a(h) - B, 0)\pi(n,h)}{\sum_n \sum_h a(h)\pi(n,h)} \quad (2)$$

which is equal to the average loss rate divided by the average arrival rate.

Validation of this methodology for analyzing a multiplexer with multiple arbitrary on/off input sources appeared in [5] and its accuracy was shown to be satisfactory.

4 Study of the Effect of Traffic Parameters on Queueing Performance

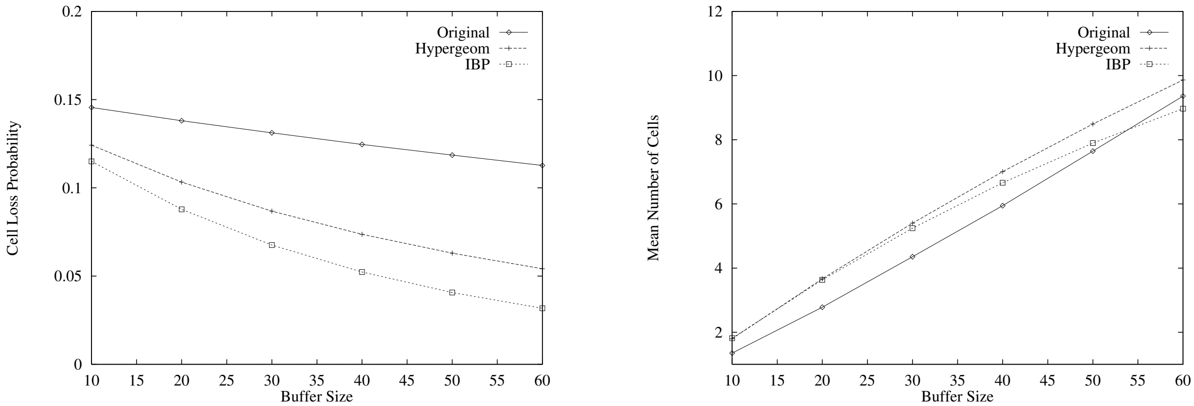
In this section, we conduct a study of the effect of various traffic parameters on the performance of the statistical multiplexer. We use the method developed in the previous section for obtaining the performance metrics.

4.1 Effect of the Distribution of the On and Off Periods on the Multiplexer's Performance

In this section, we study the effect of the distribution of the on and off periods on the multiplexer's behavior. In the literature, it is common to approximate the distribution of sojourn times in a state by a geometric or a hyper-geometric distribution. The geometric distribution characterization requires only the first moment, while the hyper-geometric distribution characterization requires the first two moments of the lengths of the periods. The question that usually arises is whether these approximations are accurate.

We consider a single server multiplexer with two homogeneous input sources and a buffer size taken from the set $\{10, 20, 30, 40, 50, 60\}$. The distribution of the on and off periods of the original input source is a mixture of two deterministic distributions. The inter-arrival time lag-1 correlation (refer to section 4.2) is equal to 0.048. The parameters of the PDF of the on and off periods are shown in table 1. The throughput of a single source is equal to 0.296, the mean on and mean off periods are 50 and 119 respectively, and the CV_{on}^2 and the CV_{off}^2 (the squared coefficient of variation of the lengths of the on and off periods) are 2.332 and 13.150 respectively. Given these values we can approximate the original source by an IBP source and subsequently by a source with a hyper-geometric distribution of the on and off periods.

In figure 4 we plot the mean number of cells and cell



(a) (b)

Figure 4: Effect of the distribution of the on and off periods on the multiplexer's performance. (a) Cell loss probability, (b) Mean number of cells.

On period		Off period	
Length	Prob.	Length	Prob.
1	0.708	20	0.950
169	0.292	2000	0.050

Table 1: Parameters of the PDF of the on and off periods.

loss probability for the original source, the single-moment approximation by an IBP source, and the two-moment approximation by a source with the hyper-geometric on and off periods. The IBP model underestimates the cell loss probability and overestimates the mean number of cells (except when the buffer size increases above 52). This suggests that the IBP is not a faithful model for the original source of table 1.

The results in figure 4, demonstrate the inaccuracy of the two-moment approximation when the buffer size is finite. The approximate source model with two-moments matching provides an underestimation of the cell loss probability and an overestimation of the mean number of cells. This shows that the two-moment approximation may not be accurate in all cases. We note here that in [16], Sohraby introduced a model for handling arbitrary on/off sources. The model gives an approximate upper bound for the cell loss probability as a function of the first two moments of the on and off periods assuming a multiplexer with an infinite buffer size. As shown in the above example, it may not be accurate to use this approximation for small values of buffer size.

4.2 Effect of the Inter-arrival Time Correlation of Traffic Sources on the Multiplexer's Performance

For an arbitrary on/off source, the lag-1 correlation coefficient of the inter-arrival time is given by Galmés [8]:

$$\phi_1 = \frac{f_{on}(1) - \frac{1}{\bar{on}}}{1 + CV_{off}^2 - \frac{1}{\bar{on}}}$$

where $f_{on}(1)$ is the probability that the on period is of length 1, $\bar{on}$ is the mean on period, and CV_{off}^2 is the squared coefficient of variation of the off period length. It is possible to identify some distributions for which the value of ϕ_1 is non-negligible. The key to obtaining such distributions is to concentrate a large portion of the probability mass at length 1 of the on period distribution, i.e. make $f_{on}(1)$ as large as possible while satisfying some of the other source characteristics. For our example here, we use the *mixture of two deterministic distributions* which is a distribution that can be of length L_1 or L_2 with probabilities p and $1-p$ respectively. We fix one of the deterministic lengths to be equal to 1 and let the other be of a variable length L. Given a particular value of $\bar{on}$ and ϕ_1 and the off period distribution, we can find values for p and L which would satisfy the given values of $\bar{on}$ and ϕ_1 using a simple enumerative algorithm.

To study the effect of the inter-arrival time correlation on the multiplexer behavior, we consider the case of two input homogeneous sources where the off period of a source has a geometric distribution with mean 92.7787 and the mean length of the on period is fixed at 50, making the source's throughput equal to 0.35. Using the mixture of two deterministic distributions for the on period, we vary ϕ_1 so that it takes values from the set $\{0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45\}$. The value of p and L satisfying the given parameters is then calculated. The cell loss probability and the mean number of cells in the multiplexer queue are shown in figure 5.

We note that by increasing the lag-1 correlation, the cell loss probability and the mean number of cells increase. As it can be seen from figure 5, the cell loss probability increases more sharply than the mean number of cells with the increase of the correlation coefficient. The mean number of cells is

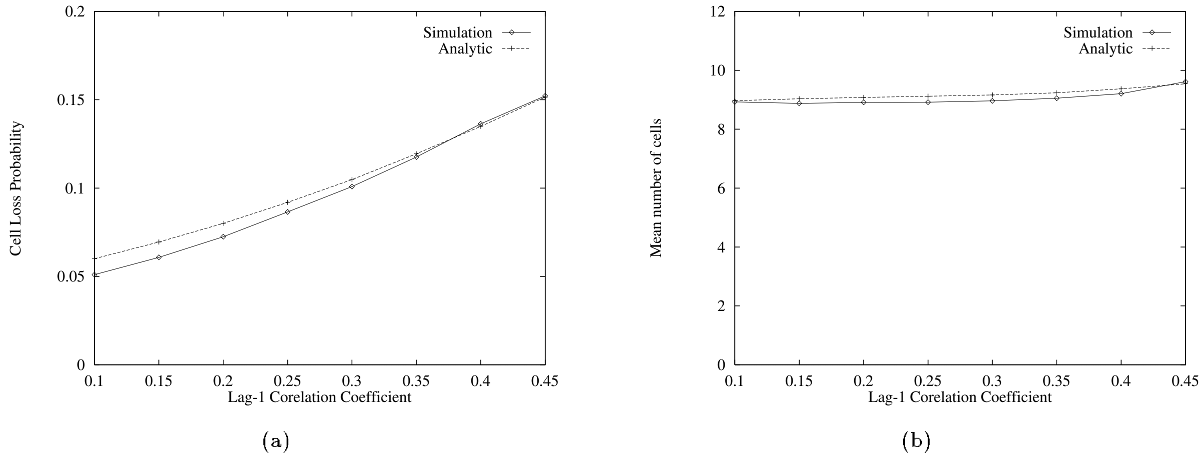


Figure 5: Effect of correlation on the multiplexer's performance. Buffer Size = 40, two input on/off sources with hyper-geometric on and off periods. (a) Cell loss probability, (b) Mean number of cells.

almost constant and increases very slowly with the increase of the correlation coefficient.

4.3 Effect of the Squared Coefficient of Variation of the on and off periods on the Multiplexer's Performance

Using the hyper-geometric distribution we study the effect of the squared coefficient of variation of the on and off periods, respectively CV_{on}^2 and CV_{off}^2, on the multiplexer's performance. By specifying the mean and squared coefficient of variation of the period length, it is possible to fit a hyper-geometric distribution given some conditions are met by the specified mean and coefficient of variation (see [4] for more details).

We consider a multiplexer with 40 cells buffer with two input sources. Each source has mean on and off periods fixed at 100 and 150 respectively. CV_{on}^2 and CV_{off}^2 take values from the set $\{1.0, 5.0, 10.0, 15.0.20.0\}$. We show the results obtained in figure 6. Note that by increasing the CV_{on}^2, while CV_{off}^2 is kept constant, both the cell loss probability and mean number of cells increase. Also, note that the rate of increase of the mean number of cells and cell loss probability when $CV_{on}^2 \in [1, 5]$ is larger than for the rest of the values. Moreover, for larger values of CV_{off}^2, the rate of increase of cell loss probability and mean number of cells as a function of CV_{on}^2 is relatively slower than for smaller values of CV_{off}^2. When increasing CV_{off}^2 while CV_{on}^2 is kept constant the cell loss probability *increases* while the mean number of cells *decreases*.

5 Conclusions

We introduced a methodology for studying finite buffer statistical multiplexers with arbitrary on/off sources input. The methodology is computationally expensive in terms of CPU and storage requirements. However, it is quite versatile and can be used to handle different types of on/off and more general traffic sources. The results obtained indicate the need for capturing the effect of the distribution of the period on call admission control and bandwidth allocation in an ATM network.

An interesting extension to this work would be to analyze the case of slow sources where the speed of incoming links to the multiplexer is less then the speed of the output link. Another natural extension would be to consider variable bit rate sources with more than two states.

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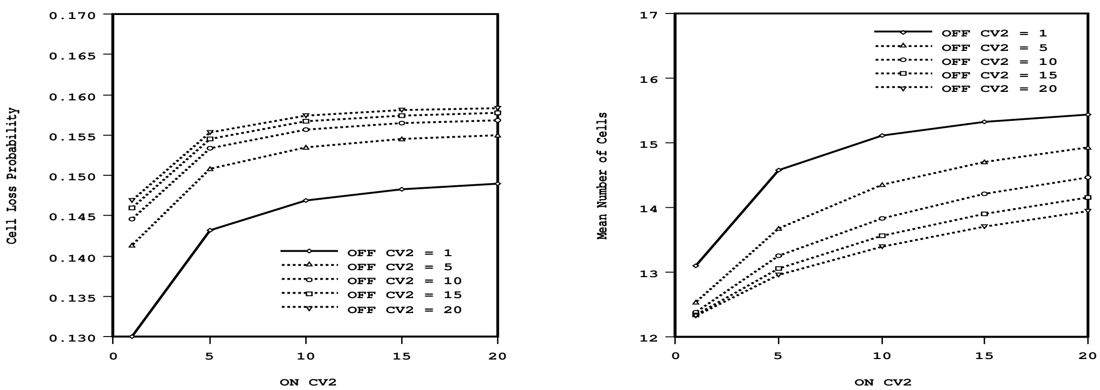

(a) (b)

Figure 6: Effect of the squared coefficient of variation of the on and off periods on multiplexer's performance. Buffer Size = 40, two input on/off sources with hyper-geometric on and off periods. (a) Cell loss probability vs. CV_{on}^2. (b) Mean number of cells vs. CV_{on}^2.

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