On the Interplay of Network Structure and Routing Strategies for Performance in Scale-Free Networks

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Abstract— **Network performance; characterized by the maximum end-to-end traffic flow the network is able to handle without overloading and has as short a route as possible between any two nodes while keeping the congestion in the network as low as possible; is an important issue in the design of Internet Service Provider's topologies. In this paper, we examine how the structural characteristics of network topologies affect the network performance and examine the interplay between structural characteristics of network topologies and routing strategies. We consider routing strategies subject to practical constraints (router technology) and economic considerations (link costs) at layer 3. We propose two new routing methods suitable for implementation in large networks and examine various routing strategies (local, global, and hybrid) with tunable parameters and explore how they can enhance the network performance. We find that there exists an optimal range of values for the tunable parameters to achieve high network performance which depends on the structural properties of the network topology. We also show that our proposal routing scheme with the minimum local information achieves high network performance.**

Index Terms— Routing strategies, network throughput, network structure.

I. INTRODUCTION

Recently, there is increased interest in studying large-scale real-world systems which include the Internet, World-Wide Web (WWW), protein-protein reactions, and social networks. The most important complex network model is the scale-free network [1] in which the nodal-degree distribution is described as

$$P(k) \propto k^{-\gamma}, \qquad (1)$$

where $P(k)$ denotes the fraction of nodes with degree k, γ is the exponent.

There are levels of connectivity ranging from the physical layer up to the application layer. The physical connectivity at lower layer is more important for router-level related issues such as network performance. Also, the existence of routing protocols sitting on top of the raw router-level connectivity is important to provide a view of network performance. The efficient throughput for communication systems is affected by the ability of the system to be aware of congestion avoidance.

It is thus of great interest to study the effects of routing strategies on traffic flow to find optimal strategies that achieve high network throughput and efficient distance communication. Ref. [2] evaluate several routing methods (min-hop, inv-cap, flow deviation method, and their proposed method) on the ISP router level topologies with different power-law degree distributions to assign link capacities and compute node loads based on these link capacities. Ref. [3] estimates the effects of variations of network structure and their proposed traffic awareness routing strategy on network capacity (total number of packets the network can handle at any given time). They build their network models using the model proposed by [4] to obtain various topologies structures with different power-law exponents.

Ref. [5] investigates numerically the scale-free network of BA model through the use of various path finding strategies. A generalized diameter is introduced and a simple strategy is suggested to yield small-world behavior. Ref. [6] proposes a routing strategy to improve the transportation efficiency on complex networks. They give a generalized routing algorithm to find the so-called efficient path, which considers the possible congestion in the nodes along actual paths.

Ref. [7] proposes a new routing strategy with a single tunable parameter which is only based on local information of network topology. They give an explanation why the delivering capacity of the network can be enhanced by choosing an optimal value for the tunable parameter.

In this paper, it is the first time to study the effects of various routing strategies on network topologies having the same node degree sequence subject to practical considerations (router technology) and economic considerations (link costs) at layer 3. We analyze the effects of variations of network structure and routing strategies on network performance.

Also, we attempt to answer the questions: how does the network throughput depend on the network topology and how does it vary with the interplay between routing strategies and network structure, and is there any relation between the efficacy of the routing strategy and network structure? We treat the nodes subject to their locations in the network topology: hosts, access routers, gateway routers, and backbone core routers.

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II. NETWORK GRAPH THEORY AND NETWORK TOPOLOGY MODELS

A. Network Graphs

A graph is a mathematical model to represent networks that have a certain structure (topology) and can have additional quantitative information (types or weights). The structure might be directed or undirected.

1) Undirected and Directed Graphs: An undirected graph *G* is defined by a pair of sets *G = (V , E)*, where *V* is a non-empty countable set of elements, called nodes or vertices and *E* is a set of unordered pairs of different nodes, called edges or links. A directed graph *D*, or digraph, is defined by a non-empty countable set of nodes *V* and a set of ordered pairs of different nodes *ED* that are called directed edges.

2) Weighted Graphs: Many real networks display a large variability in the intensity values of edges. Therefore, it is desirable to go beyond the mere topological representation and construct a weighted graph where each edge is associated with a weight representing the intensity or value of the connection.

B. Network Types

1) Homogeneous Networks: Homogenous networks mean that all nodes have a similar number of links. Homogeneity in the interaction structure means that almost all nodes are topologically equivalent, like in regular lattices or in random graphs. In these latter ones, the degree distribution is binomial or Poisson in the limit of large graph size (peaked around the average value).

2) Heterogeneous Networks: These networks, having a highly inhomogeneous degree distribution, result in the simultaneous presence of a few nodes (the hubs) linked to many other nodes, and a large number of poorly connected elements. A lightly heterogeneous network has a large number of high-degree nodes connected to a significant fraction of all nodes in the network. A highly heterogeneous network has a small number of high-degree nodes (have a large number of edges connected to these nodes) connected to a significant fraction of all nodes in the network. The degree distribution is power-law (skewed and may present heavy-tails).

3) Sparse Networks: A sparse network has an average degree that is much smaller than the size of the network, that is, $k << n$, where k is the average node degree and n is the total number of nodes.

C. Network Topology Models

We rely on measurements on data of five networks constructed explicitly to have the same node degree sequence. Fig. 1 depicts these five networks:

(a) The power-law type degree sequence of all five networks.

(b) A graph constructed from Preferential Attachment (BA model [8]):

(c) A construction based on the General Random Graph (GRG [9]) or Power-Law Random Graph (PLRG [10]) method by using the degree sequence of the BA network as the expected node degree to generate a random graph using the GRG method.

(d) Heuristically Optimal Topology (HOT [11]): constructed by using a heuristic, nonrandom, degree-preserving rewiring of the links and routers in the BA graph to produce a network having a mesh-like core with hierarchical aggregation from the edge to the core.

(e) Abilene-Inspired Topology [11]: Inspired by the publicly available actual data of the Abilene network.

(f) Sub-optimal Topology [11]: heuristically designed network that has been intentionally constructed to have poor throughput and purpose for comparison.

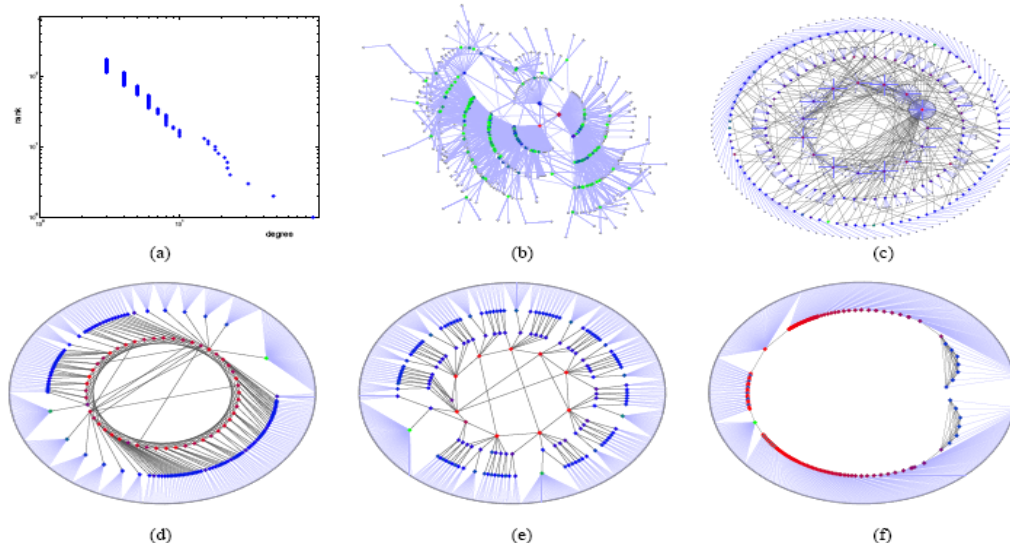These five topologies are available from [12] and constructed by Li et al. [11].

Fig. 1. Five networks having the same node degree distribution. (a) Degree distribution (degree versus rank on log-log scale); (b) BA; (c) GRG; (d) HOT; (e) Abilene; (f) SUB.

III.   NETWORK MEASUREMENT METRICS

A. Network Throughput

The network throughput is defined as the maximum proportional throughput on a network under heavy traffic conditions based on a gravity model [13]. That is, starting at the network edge we consider the demand for traffic by an access router to be the aggregate connectivity bandwidth of its end hosts (the nodes with degree one). Then, to determine the flow of traffic across the network core, we consider flows on all source-destination pairs of access routers (network edge), such that the amount of flow X_{ij} between source i and destination j is proportional to the product of the traffic demand X_i and X_j at end points i and j.

$$X_{ij} = \rho\, X_i X_j, \tag{2}$$

where ρ is some global constant and is otherwise uncorrelated from all other flows. We compute the maximum throughput on the network under the router degree bandwidth constraints (router technology),

$$\text{Maximize } Perf(g) = \sum_{ij} \rho X_{ij}, \tag{3}$$

$$\text{Subject to } RX \le B, \tag{4}$$

where R is the routing matrix (defined such that $R_{kl} = \{0;1\}$ depending on whether or not flow l passes through router k) and X is a vector obtained by stacking all the flows X_{ij} by (2), indexed to match the routing matrix R which is a routing matrix based on one of the various routing strategies introduced in section IV, and define B as the vector consisting of all router bandwidths according to the degree bandwidth constraints (router technology). Due to a lack of publicly available information on traffic demand for each end point, we assume the bandwidth demand at a router is proportional to the aggregated demand of any end hosts connected to it. In our paper, based on structural properties of the routers technology we allocate the capacities of router based on the technology constraints imposed by the Cisco 12416 GSR for all non edge routers, and by the Cisco 7500 GSR and Cisco 7600 GSR series aggregation router at the edge routers (access routers). We allocate the capacities for the links according to OC-192 for all non edge routers, OC3 and OC-24 at the edge routers.

B. Router Utilization (achieved BW)

While computing the maximum throughput of the network, we also obtain the total traffic flow through each router (router utilization). Since routers are constrained by the feasible region for bandwidth and degree, the topology of the network and the set of maximum flows will uniquely locate each router according to its degree bandwidth constraint.

C. Average Path Length

The average path length is the average shortest path length, defined as the average of the shortest distance value of over all the possible pairs of nodes in the network.

$$APL = \frac{1}{n(n-1)} \sum_i \sum_j dist_{ij}, \tag{5}$$

where $dist_{ij}$ is the shortest path in the network from i to j, and n is the number of nodes in the network.

D. Diameter Metric

The diameter Dia is defined as the maximum shortest path length in the network. That is, the diameter is the longest of all shortest paths among all possible node pairs in a graph. It states how many edges need to be traversed to interconnect the most distant node pairs.

$$Dia = Max\,(dist_{ij}) \tag{6}$$

IV.   ROUTING STRATEGIES

We examine various routing strategies and propose two new schemes. The routing methods can be classified into local, global, and hybrid (a hybrid method is a mix of local and global). In this section, we propose two routing methods that depend on the importance of a node in network communication. The first method depends on the betweenness centrality of a node while the second method is a hybrid method which is based on cost effective distance for the nodes along the path.

Our routing protocols do not use BGP protocol as we work on network topologies at router level and inside AS level. Also, we do not use OSPF protocol. Our routing protocols select the minimum path length after implementing the routing strategies for the network topologies between any two nodes in the network.

A. Global Routing Protocols

1)   Shortest Path (SP): In this routing method, it is assumed that the whole topological information is available for each node. The length of a path is the sum of cost of all edges of P. If $P = ((v_1, v_2), (v_3, v_4), ..., (v_l, v_{l+1}))$, then the length of P, denoted $L(P)$, is defined as

$$L(P) = \sum_{i=1}^{l} L((v_i, v_{i+1})) \tag{7}$$

The distance from a node v to a node u, denoted $L(v,u)$ is the minimum length (shortest path) from v to u, if such a path exists.

2)   Efficient Routing (EFFR1): Yan et al. [6] proposed a routing method to enhance the network throughput. Nodes with larger degree are more likely to bear traffic congestion, and bypassing those high-degree nodes, a packet may reach its destination quicker than taking the

shortest path and achieve high throughput. For any path between nodes i and j as: $P(i \rightarrow j) := i \equiv x_0, x_1, \ldots, x_{n-1}, x_n \equiv j$, denote

$$L(P(i \rightarrow j) : \beta) = \sum_{l=0}^{n-1} k(x_l)^{\beta} , \qquad (8)$$

where β is a tunable parameter and $k(x_l)$ is the degree of x_l. The efficient path between i and j is corresponding to the route that makes the sum $L(P(i \rightarrow j) : \beta)$ minimum.

B. Local Routing Protocols

Although the routing protocols using global topological information are generally more efficient, it is not practical for huge size communication networks. Therefore, it is more practical to study the traffic behaviors on scale-free networks based on local information.

1) Preferential Selection (PRF): Barabasi et al. [14] proposed the idea of preferential attachment as the central ingredient in order to get a power-law degree distribution using linear algebraic preferential-attachment:

$$\Pi_{j \rightarrow i} = \frac{k_i}{\sum_l k_l}, \ l \in all \ nodes \ of \ the \ network \qquad (9)$$

where the sum among all the nodes of the network. Ref. [5] uses this formula as a local strategy to find a path that connects two vertices and neglect the effect of structural dependence of the topology that chooses the node with the large connectivity that lead to traffic jam which reduces the network throughput. In our work, we use the preferential selection based on the non-linear algebraic preferential-attachment with tunable parameter which was introduced by Albert et al. [8]. We consider the local selection among all neighbors of the current node (sum among all nodes adjacent to current node only). Thus, it would be better if a routing strategy could be aware of the congestion and have some adjustability to adapt to the different congestion states of the network. The preferential selection formula is:

$$\Pi_{j \rightarrow i} = \frac{k_i^{\alpha}}{\sum_l k_l^{\alpha}} \ , \ l \in neighbor \ nodes \ of \ j \qquad (10)$$

Hence, to navigate packets, each node performs a local search among its neighbors. If a packet's destination is found within the searched area, it will be delivered directly to its target, otherwise, it will be forwarded to a neighbor j of node i according to the preferential selection formula in (10). We evaluate 20 independent network realizations for PRF method between any two node pairs and select the minimum path length from these realizations.

2) The Proposed Highly Localized Betweenness Centrality based Method (HLBC): In 1977 the sociologist Freeman defined the quantity called betweenness centrality (BC) [15] to measure the importance of a node in network communication. Employing the generating function formalism of [16], we have proposed a simple way to replace node betweenness by a local measure to restrict the length of the shortest paths considered. Instead of node betweenness, we compute the H-distance betweenness of each node: the number of shortest paths passing along it whose length is less than or equal to H. For small H, this is much faster than searching for shortest paths between all node pairs.

The Highly Localized Betweenness Centrality (HLBC) metric is defined for nodes as

$$HLBC(i) = \sum_{s \neq t} \frac{\sigma_{st}(i)}{\sigma_{st}} \ , \ s, t \in local \ subgraph \qquad (11)$$

where σ_{st} is the total number of the shortest paths from node s to node t and $\sigma_{st}(i)$ is the number of the shortest paths from node s to node t passing through node i. We assume in our work that the H-distance = 2, i.e. each node in the network has information about its first and second neighbors. Thus, to navigate packets, each node performs a local search among its neighbors. If a packet's destination is found within the searched area, it will be delivered directly to its target, otherwise, it will be forwarded to a neighbor j of node i which has the highest value of HLBC.

C. Hybrid Routing Protocols

1) The Proposed Cost Effective Distance Efficient Routing Method (EFFR2): The types of nodes typically affect the dynamics of communication during information exchange between any node pairs of the network. The proposed cost effective distance (EFFR2) depends on the important type of a node, i.e. the delivering capability for each node. The cost effective distance between node i and j is defined as

$$\cos t\_eff\_dist_{ij} = \min_P \sum_{v \in P : i \rightarrow j} D_v , \qquad (12)$$

where P is any path connecting node i to node j of the network topology, v is any node belonging to such a path. The D_v is the distance from any node to node v along the path from i to j, over all the paths connecting i and j depending on the importance type of delivering capability for each v destination node of the corresponding link (local property).

The delivering capability can be the same and then the cost of links is set proportional to the degree of the destination node of the corresponding link. Another case, the delivering capability of a node is proportional to its degree or its betweenness centrality respectively as proposed by Zhao et al. [17] to enhance the traffic capability. In this case the cost of links is set proportional to the distance of the selected destination whose degree or betweenness centrality is the largest among its neighbors of the corresponding link.

Another case which is related to the technologies used in routing equipment (technical limitation) as we use in our work and it is the major constraint affecting the delivering capability of a node that Zhao et al. [17] neglects in their proposal. Thus, when the number of router ports (degree) increases, the maximum router throughput decreases. Hence, the cost of links is set proportional to the degree of the destination node of the corresponding link.

Therefore, the $\cos t\_eff\_dist_{ij}$ is a global quantity associated with the pair i, j and is the minimum of sum of all distances D_v evaluated along the path P from i to j, over all the paths connecting i and j and the cost of links is set proportional to the degree of the destination node of the corresponding link.

V. EFFECTS OF TUNABLE PARAMETERS AND VARIOUS ROUTING STRATEGIES

A. The Impact of Tunable Parameters Settings

The measurements metrics values for PRF routing method for all our graphs are evaluated by averaging results values of more than 50 independent network realizations. It is interesting to investigate if there exists an optimal value of α and β that will make the congestion avoidance more efficient. If the optimal value exists, would it change when the congestion phase or the topology structure of the network changes?
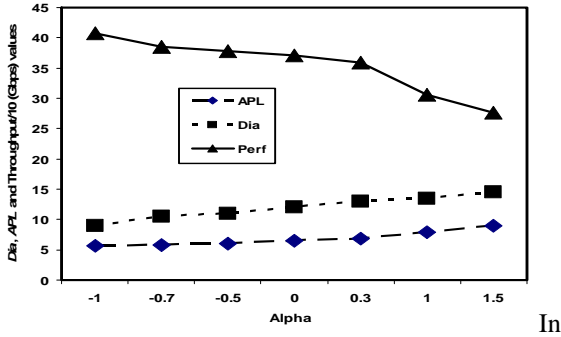

Fig. 2. Average metric values for all topologies versus α of the PRF method

As shown in Fig. 2, when α = -1, the throughput (perf) has the largest value with small *APL* and Dia and it declines as the parameter α increases with large values for *APL* and Dia. Therefore, the critical point (α = -1) can be viewed as the optimal value that should be assigned to α which makes the congestion avoidance more efficient with small Dia and *APL*. As shown in Fig. 3, with the increment of parameter β, the throughput goes up at first until the critical point is reached and then declines when the parameter β increases with values > 1. The critical point (β = 1) can be viewed as the optimal value that should be assigned to β. This value makes the congestion avoidance more efficient with small Dia and *APL*.

B. Effects of Routing Strategies and Network Structure on Router Utilization and Network Throughput

Figs 4 (b, d, f) show that the PRF method has the maximum throughput with small Dia and *APL*. The SP, EFFR1 and EFFR2 have the median values for throughput, Dia and *APL* while HLBC achieves the lowest

throughput with largest Dia and *APL*. Figs 4 (a, c, e) show that the PRF method achieves router utilization better than the other routing methods after the 95th percentile mark at the tail. Below this mark, the EFFR1, EFFR2, and SP achieve the median values, HLBC achieve larger values and PRF has the smallest values.
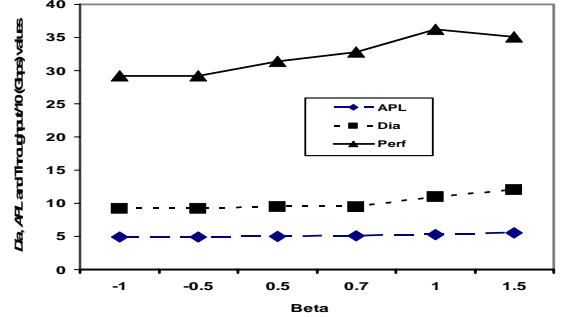

Fig. 3. Average metric values for all topologies versus β of the EFFR1 method

From Fig. 4, we can observe that the SP method concentrate traffic at the hubs of the topologies which yields low throughput. The EFFR2 and EFFR1 methods attempt to avoid hub nodes which are distributed at the router level of the network topologies; except for BA topology there is very little at the core. The HLBC method redirects the packets to the nodes having the highest local betweenness centrality (may likely choose the higher degree). Therefore, with HLBC method more nodes are likely to be congested, with EFFR1, EFFR2, and SP methods few nodes are likely to be congested, while in PRF method the chance is the smallest of the others.

The case is different with the GRG network topology. The GRG is highly heterogeneous (i.e. has some big hub nodes concentrated at the core). Fig. 4(g) shows that HLBC and SP has approximately the same distribution with very low achieved BW (bottlenecks nodes) until the 90th percentile mark and then achieve higher BW at the tail. The PRF has wider distribution than the SP and HLBC before the 98th percentile mark and then becomes the same. The EFFR2 and EFFR1 avoid the hub nodes at the core and the packets are distributed in a more dispersive fashion.

Fig. 4(h) shows that the SP and HLBC have the smallest throughput values because they select the big hub nodes at the core which results in very small throughput. However, the Dia and *APL* are small due to the effect of selecting big hub nodes which results in reaching the targets as short as possible. PRF method cannot avoid hub nodes at the core and therefore low throughput is obtained with small Dia and *APL*. The EFFR2 and EFFR1 have the largest throughput values because they redirect the packets by avoiding the big hub nodes at the core and achieve high throughput with small Dia and *APL*.
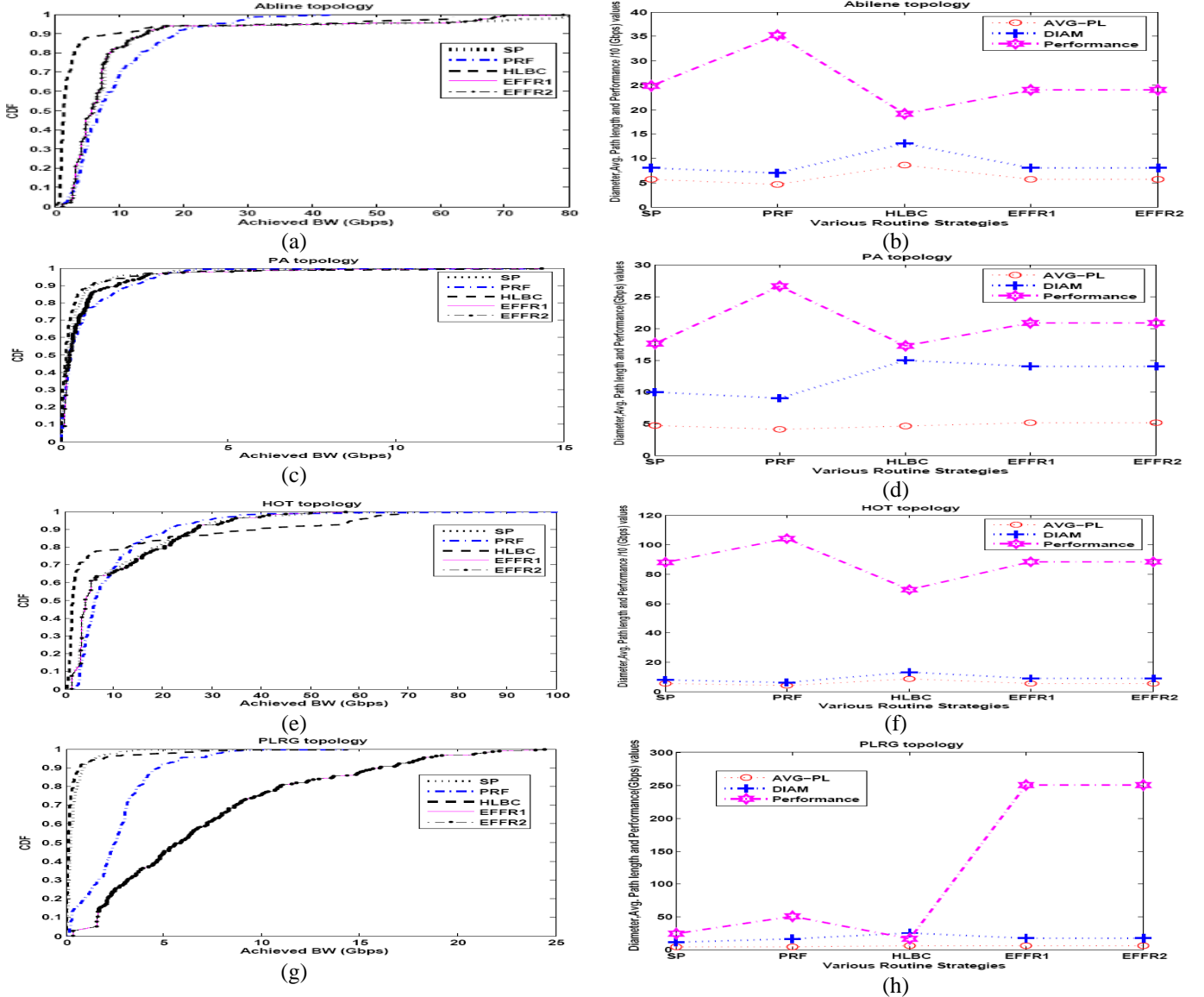
Fig. 4. Router utilization (left) and diameter, *APL*, performance (throughput) at (right) for the topologies (a,b) Abilene, (c,d) BA, (e,f) HOT, and (g,h) GRG.

From the above observations, it is obvious that the network throughput depends mainly on the network topology structure and the routing method applied to this network topology. Our results for all routing strategies show that the topologies for the BA and the GRG achieve poor throughput because these degree-based models have the highly connected "hubs" that create low-bandwidth bottlenecks. However, in the Abilene and HOT topologies with mesh-like core, the aggregated traffic is dispersed across multiple high-bandwidth routers and therefore achieve largest network throughput (the throughput values for HOT and Abilene divided by 10 to adjust the graph scale). Table I shows the throughput normalized to total capacity for every topology with all the routing methods. Therefore, from Table I and the above results,.the order of the best routing method throughput is PRF > EFFR2 and EFFR1 > SP > HLBC for all topologies except for GRG the

order is EFFR2 and EFFR1 > SP > HLBC > PRF. From the above observations, we can provide some useful insights about the strong correlation between the routing methods efficiency and the network structure topology. Thus, we can give an indication about which routing method is suitable for specific network structure. Table II shows these indications.

TABLE I
THROUGHPUT NORMALIZATION TO TOTAL CAPACITY FOR EACH TOPOLOGY

Topology	Routing Methods				
	SP	PRF	HLBC	EFFR1	EFFR2
ABILENE	0.4593	0.5893	0.5293	0.4693	0.4693
BA	0.2855	0.3027	0.2881	0.2704	0.2704
GRG	0.2589	0.2045	0.2569	0.2763	0.2763
HOT	0.6895	0.8012	0.6141	0.6833	0.6833
SUB	0.1178	0.1268	0.1193	0.1178	0.1178

TABLE II
THE RELATION BETWEEN ROUTING METHODS AND NETWORK TOPOLOGIES STRUCTURE

Routing Method	Network Topology Model		
	Heterogeneous	Homogenous	Sparse
PRF	Suitable for lightly heterogeneous. Suitable for highly heterogeneous except when there are highly-connected hub nodes at the central core.	Not suitable	Not suitable
HLBC	Suitable for lightly heterogeneous Not suitable for highly heterogeneous except when the highly connected nodes become powerful and efficient. Suitable for highly heterogeneous when considering efficient distance communication	Not suitable	Suitable
EFFR2	Suitable when the importance type of a node does not depend on its degree.	Not suitable	Suitable when the importance type of a node does not depend on its degree.
EFFR1	Suitable and better for highly heterogeneous than for lightly heterogeneous.	Suitable only when $\beta = 0$	Not suitable when $\beta > 0$
SP	Suitable for lightly heterogeneous but not suitable for highly heterogeneous.	Suitable for all homogeneity distributions	Suitable

VI. CONCLUSIONS

Our results show that the physical connectivity and the existence of routing protocols sitting on top of the raw router-level connectivity is important to provide a view of network throughput. It is found that there is a strong correlation between the routing efficiency and the network structure topology The effect of the same routing mechanism for traffic flow varies for different topologies depending on the underlying network structure (different degree distribution or they have the same degree distribution by our work). Also, the effect of various routing strategies varies for traffic flows for the same network topology structure. Our observations show that the throughput of the communication systems can be improved by implementing the appropriate routing method if it will be costly or even impossible to change the underlying network structure for real networks.

Hence, various routing strategies are proposed and it is found that the network throughput is greatly enhanced when the routing strategies are designed in a congestion-aware manner (EFFR2, PRF). Therefore, we give an indication about which routing method is suitable for specific network structure. Our results show that SP is not necessarily an efficient routing method for network topologies having various degree distributions.

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