A New Small-World Rewiring Method for Construction of Heuristically Optimized Trade-Offs Router Network Topologies

Walid K. Ghamry
National Research Center
Cairo, Egypt.
wghamry@yahoo.com

Khaled Elsayed
senior member IEEE
Cairo University
khaled@ieee.org

Ameen Nassar
Cairo University

***Abstract*—The structure of complex networks has been widely described as scale-free networks generated by the preferential attachment model. However, these models do not take into account the more detailed description of the underlying topological physical structure observed in real networks. In this paper, we propose a new simple synthetic model of the Internet's router-level topology based on Heuristically Optimized Tradeoffs (HOT) concept. The trade-offs between system throughput and the technological and economic constraints that are crucial when designing the synthetic system. We propose a new edge rewiring/addition process for a small-world model with tunable parameters to address the aforementioned issues for high variability in backbone core structure connectivity. Our proposal approach can reproduce the low-likelihood topology metric (topological disassortativity of real networks) and can satisfy the small-world effect at the same time to achieve reasonably "good" network functional requirements.**

Keywords-component; HOT design; network throughput; likelihood; small-world structure

I. INTRODUCTION

Recently, there is increased interest in studying large-scale real-world systems which include the Internet, World-Wide Web (WWW), protein-protein reactions and social networks. The most important complex network model is the scale-free network [1] in which the nodal-degree distribution is described as

$$P(k) \propto k^{-g}, \quad (1)$$

where $P(k)$ denotes the fraction of nodes with degree k, g is the exponent.

Understanding the large-scale structural properties of the Internet is critical for network designers and can have a dramatic impact on its throughput [2]. The large-scale topological structure of the Internet can be made on routers in the router-level graph [3] or entire subnetworks (Autonomous Systems) in the AS-level graph [4]. The general power of an optimization-based approach to understanding power-laws in complex systems has been documented as part of the so-called HOT concept, for Highly Optimized Tolerance [5] or Heuristically Optimized Tradeoffs [6]. The first explicit attempt to cast topology design, modeling, and generation as a HOT problem was by Fabrikant et al. [6]. They proposed a model of incremental access network design that optimizes a tradeoff between connectivity distance and node centrality.

Li et al. [7] propose a complimentary approach of combining a more subtle use of statistics and graph theory with a first-principles theory of router-level topology that reflects practical constraints and tradeoffs. They propose their heuristically optimal topology (HOT) to reflect its consistency with real design considerations and address the aforementioned issues for high variability in end-user connectivity while keeping the core structure unchanged to achieve reasonably "good" throughput.

In this paper, we introduce simple synthetic model of the Internet's router-level topology using the gravity method of [8] and the notion of "first-principles approach" based on HOT concept that introduced by [7]. HOT stands for Heuristically Optimized Tradeoffs where the tradeoff is between system throughput and link cost subject to router technology constraints that are crucial when designing the synthetic system model.

Our approach for generating realistic router level topologies is based on the assumption that topology creation within an AS must regard not only market demands (link costs and hardware constraints) but introduce power-law degree distribution too. We study the effects of core structure variability on network behavior by proposing a new edge rewiring/addition process of a small-world model with tunable parameters to construct a sparse mesh-like core topology. We design a new small HOT design at router level and address the aforementioned issues for high variability in backbone core structure connectivity while keeping the remaining nodes sequence degree connectivity at access and gateway routers unchanged.

II. THE PROPOSED HOT DESIGN METHOD

A. *Functional Requirements*

The primary purpose for designing a network is to provide connectivity, carry effectively a projected overall traffic demand, and consider a first-principles theory of router-level topology that reflects practical constraints and tradeoffs. These observations yield different metrics for designing our proposed HOT network such as network throughput, router utilization, network likelihood, average path length, diameter, and mean clustering coefficient. For our designing concepts, we consider the graph g is simple (i.e. self-connections and duplicate edges are not allowed), connected, and whose links and nodes are annotated and specify link bandwidth and router type.

1) Network Throughput Metric: The network throughput is defined as the maximum proportional throughput on a network under heavy traffic conditions based on a gravity model [8]. We consider the demand for traffic by an access router to be the aggregate connectivity bandwidth of its end hosts.

We consider flows on all source-destination pairs of access routers such that the amount of flow X_{ij} between source *i* and destination j is proportional to the product of the traffic demand X_i and X_j at end points *i* and *j*.

$$X_{ij} = r \; X_i X_j , \quad (2)$$

Suppose that there is a network where B is the vector of all router bandwidths according to the degree bandwidth constraint (figure 1), R is a routing matrix obtained using standard shortest path routing, and X is the vector of the traffic demands. Therefore, the Linear Programming (LP) model using tomogravity method [9] can be constructed with the following objective function and constraints to compute the maximum proportional throughput on the network under the router degree bandwidth constraint,

$$\text{Maximize } Perf\,(g) = \sum_{1 \le k \le |x|} r\, X_k , \quad (3)$$

$$\text{Subject to } \sum_{k=1}^{f} R_{rk} X_k \le B_r ,\; r = 1, 2, ..., m , \quad (4)$$

where m is the number of routers, X_k is the OD pair of (2), f is the number of flows on a router, routing matrix R is composed by {0,1} with rows representing the network routers and columns representing the OD pairs, and r is a constant ratio representing the proportional level among all flows to maximize the throughput while minimizing all the flows pass through the routers to avoid overloading.

2) Likelihood-Related Topology Metric (s-metric): We consider the s-metric as introduced in [10]. Let $G(D)$ denote the set of all connected simple graphs having the same degree sequence D. For any graph g having fixed degree sequence, we define the metric

$$s(g) = \sum_{(i,j) \in E} d_i d_j , \quad (5)$$

where d_k denotes the degree of vertex k and $s(g)$ depends only on the graph g. Hence, we define the network likelihood of g as the normalized metric

$$S(g) = s(g) / s_{\max} , \quad (6)$$

where $s_{\max}$ is the maximum value of $s(g)$ among all $g \in G(D)$. There is a direct relation between $S(g)$ and the measure *r(g)* of assortativity in networks as introduced by Newman in [11]. Thus, the metric $S(g)$ measures the interconnectedness between the hub nodes of a graph, the extent to which the graph g has a "hub-like" core (i.e. high connectivity nodes forming a cluster in the center of the network).

3) Router Utilization (achieved BW): While computing the maximum throughput of the network, we also obtain the total traffic flow through each router (router utilization). Since routers are constrained by the feasible region for bandwidth and degree, the topology of the network and the set of maximum flows will uniquely locate each router within the feasible region.

4) Average Path Length (APL) Metric: The average path length is the average shortest path length, defined as the average of the shortest distance value of over all the possible pairs of nodes in the network.

$$APL = \frac{1}{n(n-1)} \sum_i \sum_j dist_{ij} , \quad (7)$$

where $dist_{ij}$ is the shortest path in the network from i to j, and n is the number of nodes in the network.

5) Diameter Metric: The diameter Dia is defined as the maximum shortest path length in the network. That is, the diameter is the longest of all shortest paths among all possible node pairs in a graph. It states how many edges need to be traversed to interconnect the most distant node pairs.

$$Dia = Max\,(dist_{ij}) , \quad (8)$$

6) Clustering Coefficient Metric: A common feature of many complex networks is clustering. Node clustering is a measure of how well the neighbors of a given node are locally interconnected (how close a node's neighbors are to forming a clique). Node clustering c_i is defined in [12] as the ratio between the number of edges E_i among the neighbors of a node i of degree k_i and the maximum number of possible edges $k_i(k_i - 1)/2$. The node clustering coefficient is defined as

$$c_i = \frac{2\,E_i}{k_i(k_i - 1)} \quad (9)$$

The clustering coefficient of the whole network $\langle c \rangle$ is the average of c_i over all the nodes in the network

$$\langle c \rangle = \frac{\sum_{i=1}^{NN} c_i}{NN} , \quad (10)$$

where NN is the total number of nodes. The clustering coefficient is practical metric that measures the local robustness in the graph: the higher the local clustering of a node, the more connected are its neighbors, thus increasing the path diversity locally around the node.

B. Designing Concepts

The perspective of an ISP in building a network topology is driven by various factors, we focus on some important factors.

- The need to minimize the long distance link costs means that it is driven to aggregate traffic from its edges to its core.
- The design of its topology, particularly in the core, must conform to the technology constraints inherent in routers.

- The network should have good throughput, measured in terms of its ability to carry large volumes of traffic in a fair manner.
- The communication between the resource components of a network topology must be efficient and as quick as possible.
- The robustness of the network by increasing the path diversity locally around the node (high clustering coefficient) and achieving small vertex cover (small likelihood *s*-metric).

C. Construction of the Proposed HOT Design

Our proposed HOT design under gravity model is straightforward and follows by inspection in a highly abstracted way the design of real networks.

- The degree one nodes are designated as end-user hosts and placed at the periphery of the network and the number of edge routers placed at the edge of the network follows according to the degree of each gateway connected to it and has many low-speed connections.
- The connections among gateway routers are adjusted such that their aggregate bandwidth to a core router is almost equally distributed and these are the next level from the backbone core.
- The backbone core is designed as a small-world network with a sparse mesh-like topology; i.e. have high capacity but have only a few high-speed connections. This is constructed by positioning a backbone core built on a low-dimensional regular lattice and then adding or moving edges to create a low density of "shortcuts" that join remote parts of the lattice to one another.

In our design, based on structural properties of the routers technology we allocate the capacities of router based on the technology constraints imposed by the Cisco 12416 GSR for all non edge routers, and by the Cisco 7500 GSR and Cisco 7600 GSR series aggregation router at the edge routers (access routers).

D. Implementing the Regular Ring Lattice with Different H Neighbor Ring-Induced Distance (H-NRID) Parameter.

A regular one dimensional lattice with periodic boundary conditions is generated (i.e. a ring of routers). Every router is connected to its first k neighbor routers closest to it ($k/2$ on either side), where $\langle k \rangle$ is the average degree of the core lattice router. A ring lattice entails a natural notion of distance, which is distinct from that of shortest path. A pair of routers in a ring lattice are close according to the H-NRID (lattice spacing away) if the (shortest) arc that connects them, along the circle outlined by the ring, is small, i.e., it crosses few other routers. The H-NRID can vary from 2, 3, … INT$(N/2)$ and N is the number of core routers in a ring lattice.

In our implementation design, we choose $\langle k \rangle$ = 4, H-NRID starting with 2, and N with various numbers (30, 50, and 70 respectively) to study the effect of variable core structure numbers in network functional requirements while keeping the gateway routers, edge routers and hosts remain unchanged. Table I show the resulting network topologies statistics for the proposed HOT design with variable regular lattice core structure. The table shows that the power-law exponent, number of nodes, number of links, and number of internal routers increase according to the increase in core structure routers. Now we will study the effect of variability in H-NRID parameter for all the three topologies on network functional requirements. Fig. 1 and Fig. 2 show *Perf* (g) (throughput), $\langle c \rangle$ (mean clustering coefficient), and $S(g)$ (likelihood s-metric) denoted by Perf, CC, and LH respectively under various H-NRID values for various regular lattice core structure numbers (30, 50, 70).

In each case, a striking contrast is observed by strong relation (correlation) between the network throughput and network likelihood for all three proposed HOT regular models. In each case, with the increment of tunable parameter H-NRID, throughput goes up while the likelihood goes down at first until the critical point is reached.

TABLE I. THE PROPOSED HOT TOPOLOGIES WITH VARIOUS CORE STRUCTURE NUMBERS

Topology with core routers number	**nodes**	**links**	**Internal routers**	**End hosts**	**Power-law exponent (g)**
			Gateway and edge		
Reg. 30	930	1020	242	658	-1.5639
Reg. 50	950	1100	242	658	-1.5752
Reg. 70	970	1180	242	658	-1.5921

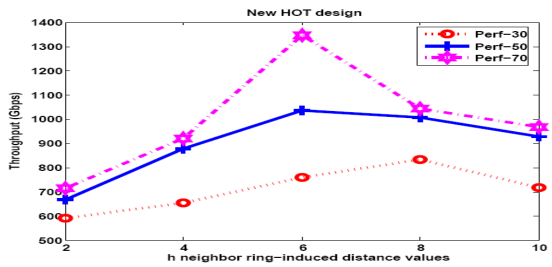

Figure 1. Proposed HOT throughput

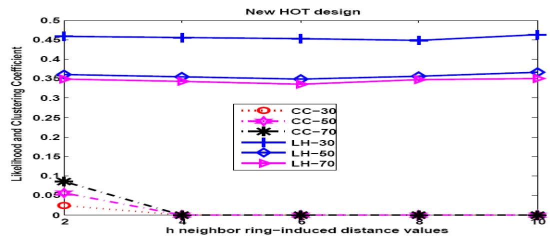

Figure 2. Proposed HOT $\langle c \rangle$ (dashed curves) and $S(g)$ (solid curves)

The critical point is H-NRID = 6 for regular-50 and regular-70, H-NRID = 8 for regular-30. Therefore, the throughput declines while the likelihood increases with the increment of H-NRID parameter. The throughput for the proposed HOT regular-70 is the largest one as its likelihood is the smallest one, then proposed HOT regular-50, and proposed HOT regular-30. The reason for these results observations due to the change of degree distribution for the remaining gateway and access routers from narrowly distributed for regular-30 to wide distribution for regular-50 to more wide distribution for regular-70 to achieve the same number of end hosts, gateway routers, and edge routers. Accordingly, the likelihood is high for regular-30 and decreases from regular-50 to regular-70. Fig. 2 shows the clustering coefficient increases from regular-30 to regular-50 to regular-70 when H-NRID = 2, which is expected: the more the links, more the clustering.

E. Implementing our Proposed Edge Rewiring/Addition Process with Different Probability P_{ws} Parameter

In our design for the second step to generate small-world core network, we implement our proposed edge rewiring/addition process by performing one of the following two operations with tunable parameter P_{ws} :

- **Edge Rewiring**: The small-world model is created by the original algorithm as proposed in [12], by taking a small fraction of the edges in the regular lattice core graph and "rewiring" them. The rewiring procedure involves going through each edge in turn and moving one end of that edge to a new location with tunable probability parameter P_{ws} .
- **Edge Addition**: Another variant of the model that has become popular was proposed by [13,14]. The small-world model is created by adding a small fraction of the edges in the regular lattice core graph and no edges are rewired. Instead "shortcuts" joining chosen vertex pairs are added to the low-dimensional lattice with tunable probability parameter P_{ws} that governing the density of these shortcuts and is defined as the probability per edge on the underlying lattice of there being a shortcut anywhere in the graph.

Hence, in each case there is a mean total number of $P_{ws}\ N\ k\ /\ 2$ rewired/added edges introduced in the proposed HOT graph, such that self-connections and duplicate edges are excluded, at its backbone core. In both cases, the vertex pair to be rewired or added as a shortcut are chosen randomly; as we proposed; in the way that decreases *APL* and increases $\langle c \rangle$ and in the same time connect *s*(*g*) in several ways to reproduce the topological low-likelihood s-metric where high degree routers preferably connected to lower-degree routers (disassortativity) for the sub-graph of the core topology with the effect that local rewiring/addition process has on the global structure of graphs in the set *G*(*D*). Our proposed method is processed as follows:

Based on the degree distribution, all routers at the backbone core are labeled and arranged in two lists according to their degrees (high or low). We set a threshold value TH_{lh} to distinguish between higher and lower degree values and thus, the higher degree routers have a degree $\geq TH_{lh}$, and lower degree routers have a degree $< TH_{lh}$. With tunable parameter P_{ws} , we rewire/add $P_{ws}\ N\ k\ /\ 2$ edges. For this we randomly select a router i with high degree from its list and remove an edge l_{ij} connected to it for case 1 and without removing any edge for case 2, then replacing it with a new edge $l_{ij'}$ that connects router i with router j' chosen randomly from the lower degree list.

III. Comparing the Functional Requirements of the Proposed HOT under our Proposed Approach

In our implementing design we choose TH_{lh} = 5, $P_{ws} \geq 2\ /\ Nk$ as a starting tunable parameter value guaranteeing the existences of at least one rewire/shortcut edge introduced. Fig. 3 and Fig. 4 show a strong relation between the network throughput and network likelihood for all the proposed HOT topologies. In each case, with the increment of tunable parameter P_{ws} , the throughput goes up while the likelihood goes up at first according to the edge rewiring/addition process that change the degree distribution of the backbone core structure until the critical point is reached (P_{ws} = 0.2 for edge addition, and P_{ws} = 0.26 for edge rewiring) and then the throughput declines while the likelihood increases significantly with edge addition process, and with very small values for edge rewiring process.

Fig. 5 shows that the *APL* and *Dia* for all the topologies with edge rewiring/addition process decreases with the increase of P_{ws} values, but in edge addition process has smaller values than the edge rewiring process because it is likely to connect widely separated parts of the regular. Fig. 4 and Fig. 5 show a strong positive correlation between $\langle c \rangle$ and and the *APL*. Thus, with the increment of P_{ws} values, we decrease the *APL* and increase the mean clustering coefficient $\langle c \rangle$.

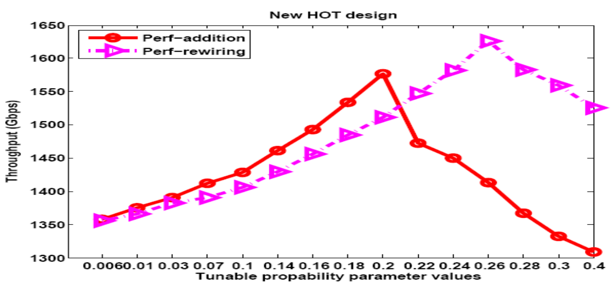

Figure 3. Proposed HOT $Perf\ (g)$ for rewiring/adding edges with various P_{ws} tunable probability parameter

Thus, we make evidence that our approach design achieves the small-world effect for our proposed HOT topologies design. Fig. 6 shows the router utilization under edge rewiring/addition process at the maximum throughput and it is found that more routers are likely to be congested in edge addition process than few routers are likely to be congested in edge rewiring process. Fig. 7 shows our final proposed HOT model's mesh-like core topology design approach by edge rewiring process and functional requirements are shown in table II.

IV. Conclusions

Studying the topological structure of complex networks has been one of the most fundamental steps to gain a basic understanding of certain aspects of real-world phenomena. Since the Internet is a collection of thousands of smaller networks, there is no single place from which one can obtain a complete picture of its topology. We show that networks could be designed for a purpose and the Internet topology can be understood in terms of the tradeoffs between throughput and the technological factors constraining the design.

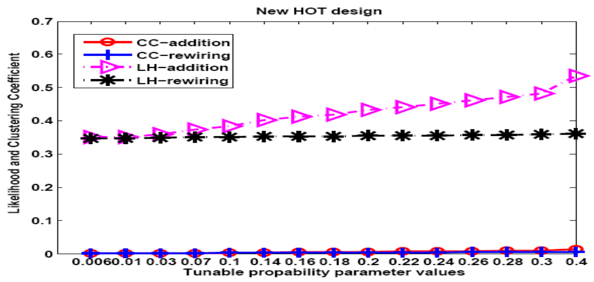

Figure 4. Proposed HOT $\langle c \rangle$ and $S(g)$ for rewiring edges (dashed curves) and adding edges (solid curves)

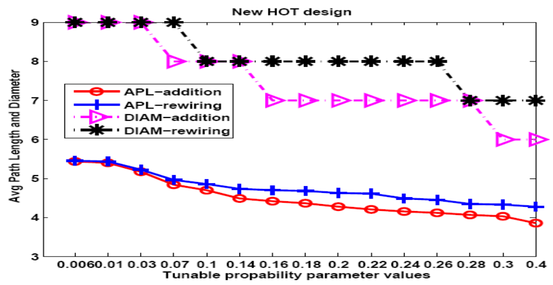

Figure 5. Proposed HOT *APL* , and *Dia* for rewiring edges (dashed curves) and adding edges (solid curves)

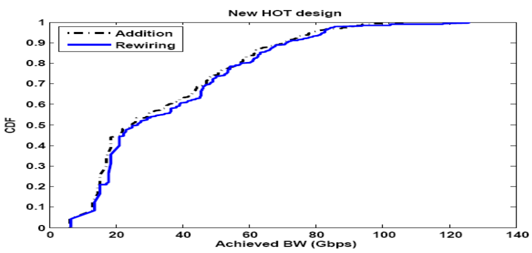

Figure 6. New HOT router utilization for rewiring/adding edges at P_{ws} = 0.26/0.2 tunable probability parameter

TABLE II. The functional requirements values for our proposal HOT topology

$Perf(g)$	$\langle c \rangle$	$S(g)$	**APL**	**Dia**	P_{ws}	**H-NRID**	$(g\ )$
1625 Gbps	0.0052	0.3568	4.431	8	0.26	6	-1.52

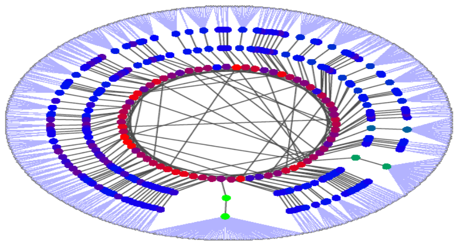

Figure 7. The final new HOT topology structure of our proposal approach

.. Our proposed edge rewiring/addition process for small-world model plays an important role to build network topologies in several ways to reproduce low-likelihood s-metric values that achieves high network functional requirements (throughput, robustness).

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